Specification of DSGE Models: A Business Cycle Accounting Application for Colombia*

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Abstract

In this document we apply the Business Cycle Accounting (BCA) procedure in order to study which mechanisms are more relevant to interpret the dynamics of the Colombian GDP between 1994 and 2009. The neoclassical growth model with endogenous variable capital utilization of Cavalcanti et al. [2008] is used as a benchmark model. This reference model can be seen as a reduced form of a greater number of DSGE models with different frictions commonly used in the literature. This equivalence allows us to assess the importance of each of those detailed models in explaining the fluctuation in the economic activity. The results suggest that in order to achieve better predictions for the GDP, macroeconomic models should include distortions that alter the consumption-savings decisions and the labor supply of the households. These distortions can be obtained with the inclusion of capital and labor wedges on the benchmark model. Furthermore, frictions related to movements in the total factor productivity tend to overestimate periods of persistent economic downturn such as the one registered between 1998 and 2000 after the Asian crisis.

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1 Introduction

"That these predictions were wildly incorrect and that the doctrine on which they were based is fundamentally flawed are now simple matters of fact, involving no novelties in economic theory. The task now facing contemporary students of the business cycle is to sort through the wreckage, determining which features of that remarkable intellectual event called the Keynesian Revolution can be salvaged and put to good use and which others must be discarded" [Lucas and Sargent, 1979]

During the last decades, the economic science has seen a great increase in the number of dynamic stochastic general equilibrium models (DSGE) that incorporate jointly or independently the role of aggregate demand, its relation with nominal rigidities and/or wages, the role of technological shocks, labor markets and the strategy to model unemployment, the credit markets and financial systems. All of them with the only purpose of approximating quantitatively and qualitatively in the best possible way the behavior of the business cycles. For emerging economies, it is worth mentioning the work of Mendoza [1991, 1995], Rebelo and Vegh [1995], Mendoza and Uribe [2000], and Neumeyer and Perri [2005] who have concluded that the business cycles in developing countries can be explained by terms of trade shocks, foreign interest rate shocks, the existence of distortions in the international capital markets or by intertemporal distortions created with the exchange rate stabilization programs.

In view of all this proposals on how the business cycles are generated, it is relevant to ask which is the most appropriate. Regarding this matter, Chari et al. [2007] point out that the flexibility of DSGE models to incorporate both neoclassical and New Keynesian ideas creates a lot of problems for researchers since they face the dilemma of where and which frictions to introduce in such a way that the simulated outcomes replicate the real data. For these authors, the Great Depression still remains unexplained. Even though the literature has offered a great number of theories to explain the drop and slow recovery of output, there is no consensus about the forces that lead to that economic event.

To overcome this problem, some economists have used tools from time series analysis such the VAR and SVAR models in the design of DSGE models [1]. However, authors like Chari et al. [2008] have criticized their use in the study of economic business cycles due to the lack of economic theory behind them. In this matter, Chari et al. [2007] (CKM hereafter) propose a diagnosis method to guide researchers on the type of frictions that are quantitatively relevant prior to the construction of a detailed DSGE model with the only objective of obtaining a better approximation to the observed business cycles.

The method, called business cycle accounting, relies on an equivalence result according to which a closed economy neoclassical growth model with time varying correlated disturbances, called benchmark model, corresponds to a reduced form of a great number of DSGE models that add different frictions, called detailed models.

[1] A detailed and advanced study of this subject can be found in Canova [2007].

The disturbances introduced in the benchmark model are called wedges and represent distortions within an economy that operates in perfect competition, which affect the productivity and the efficient utilization of the production factors (Cavalcanti, 2007). As stated by Cavalcanti et al. [2008], these disturbances are called wedges in the sense that "*they measure how far is the real economy from a closed, non-stochastic and perfectly competitive economy"*. In other words, the method provides a way to give an structural interpretation to the residuals resulting from the fitted macroeconomic model to the observed data [2]. As is shown later, the fact that the benchmark economy is closed does not imply that detailed open economy models, appropriate for economies like Colombia, cannot be represented by the former.

The first step in the accounting procedure is to construct time series for each of the wedges included in the benchmark model. In their original work, CKM define four wedges: an efficiency wedge, a labor wedge, an investment wedge and a wedge related to the government consumption, which all together can be interpreted as productivity, taxes on labor and investment, and government expenditures, respectively. The observed time series for the first three wedges are obtained from the first order conditions. But note, that for the investment wedge, which comes from the stochastic Euler intertemporal condition, assumptions about the way the agents form their expectations have to be done. This has generated a lot of discussion around the method since the results may not be robust to the assumptions made (Christiano and Davis, 2006), and the equivalence result between the wedges and the detailed models may not be so general (Bäurle and Burren, 2007). With the idea of avoiding the need to make assumptions about expectations formation, Cavalcanti et al. [2008] substitute the investment wedge by a capital wedge, which can be interpreted as a tax on capital, and include in the benchmark model the capital utilization as an endogenous variable chosen by the household. Regarding the first modification, Bäurle and Burren [2008] show that this does not affect the results of the method since the equilibrium allocations from the benchmark model are identical under both specifications.

Once the time series for the wedges are obtained, the relative importance of each of them in the dynamics of the observed macroeconomic variables is evaluated. This is done by simulating the benchmark model using a subset of the wedges, that is, by feeding the model with each of them separately or in groups. This allows us to study the proportion of output movements that can be explained by each of the wedges. By construction, when the four wedges are included simultaneously, the observed and simulated time series are equal at each point of time. The simulated path obtained only including the efficiency wedge, CKM call it "*efficiency wedge alone economy*". Similarly, it is possible to define, in the case where only individual wedges are used, the "*labor wedge alone economy*", "*capital wedge alone economy*" and "*government wedge alone economy*". In the case of combinations it is possible to define "*efficiency and labor*

[2] This idea is not new in economics. As mentioned by Christiano and Davis [2006], the Solow residual is another example.

alone economy", or the *"capital and labor alone economy"*, among others.

Finally once the relevance of the wedges is assessed, it is possible to use the equivalence result to find possible explanations for the observed behavior. In this sense, the BCA method is a diagnosis tool in the design of appropriate DSGE models: recommend detailed economies that better fit the behavior of the wedges included in the benchmark economy.

It is important to clarify that the method does not identify a detailed model in particular in order to replicate the stylized facts of the economy. On the contrary, since each of the wedges could be originated by different types of models, the BCA methodology provides a guide to what "class" of models can do a better work for studying the observed data. In this sense, this document aims to asses, identify and quantify, under an unified framework, the relative importance of different kind of distortions in such a way that it contributes in the construction of more data consistent DSGE models, and to help direct the macroeconomic discourse and the policy discussions towards the more relevant wedges.

The rest of the article is structured as follows. In Section 2 the benchmark model is described. In Section 3 some of the possible equivalence results between the benchmark and the detailed economies are presented. Later, in Section 4 the benchmark model is calibrated, the wedges constructed and the quantitative analysis done. Finally, Section 5 concludes.

2 Benchmark model

The benchmark model corresponds to a closed economy discrete-time version of the neoclassical growth model with time varying wedges. In this economy agents maximize their life time expected utility by choosing optimal paths for consumption, c_t, hours worked, l_t, investment, x_t and capital, k_{t+1}. Following Cavalcanti et al. [2008], every period the agents also choose what proportion of their accumulated capital stock want to rent to the firms. Let μ_t be the effective capital utilization rate in period t. Capital depreciation will be an increasing function of the utilization in order to capture the fact that a higher use of the capital implies a greater deterioration of it.

The representative agent problem is:

$$\max_{\{c_t, l_t, x_t, k_{t+1}, \mu_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, o_t\right), \quad \beta \in (0,1)$$

subject to

$$
\begin{aligned}
c_t + x_t &= (1-\tau_{lt}) w_t l_t + (1-\tau_{kt}) r_t (\mu_t k_t) + tr_t \\
k_{t+1} &= (1-\delta_t) k_t + x_t \\
\delta_t &= \bar{\delta} \exp (a (\mu_t - \bar{\mu})) \\
l_t + o_t &= 1 \\
k_0 &> 0 \text{ , given}
\end{aligned}
$$

where c_t is the consumption level, k_t is the capital stock, x_t the gross investment, all in per capita terms; w_t is the real wage paid to workers, r_t is the rental rate of capital, β is the subjective discount factor and tr_t are per capita lump-sum transfers that the agent receive from the government; δ_t is the time-varying depreciation rate of capital which is a function of the capital utilization rate, μ_t; $\bar{\delta}$ is the average depreciation rate, a measures the strength with which the depreciation rate responds to changes in the utilization of capital, and $\bar{\mu}$ is the average capital utilization. o_t is the fraction of time the agent devotes to leisure in period t. Hence, l_t is the fraction of time he chooses to work. Finally, τ_{lt} and τ_{kt} define the labor and capital wedges which can be interpreted as taxes on labor and capital respectively.

On the other hand, firms have access to a constant return to scale production function of the following type:

$$
y_t = A_t F \left((\mu_t k_t) , (1+\gamma)^t l_t \right)
$$

where y_t is the per capita level of output that can be produced with the effective level of capital rented from households $(\mu_t k_t)$ and labor l_t which grows at an exogenous and constant rate $(1+\gamma)$. A_t corresponds to the efficiency wedge. Firms must choose the quantity of capital and labor that maximizes their profits:

$$
\max_{\{k_t, l_t\}_{t=0}^{\infty}} \quad A_t F \left(\mu_t k_t, (1+\gamma)^t l_t \right) - r_t \mu_t k_t - w_t l_t
$$

Finally, it is assumed that the government follows a balanced budget in every period:

$$
g_t + tr_t = \tau_{lt} w_t l_t + \tau_{kt} r_t \mu_t k_t
$$

where g_t are the government expenditures.

The first order conditions from the household problem are:

$$
\begin{aligned}
\frac{u_l\left(c_t, 1-l_t\right)}{u_c\left(c_t, 1-l_t\right)} &= -\left(1-\tau_{lt}\right) w_t \\
\left(1-\tau_{kt}\right) r_t &= a\bar{\delta} \exp\left(a\left(\mu_t-\bar{\mu}\right)\right) \\
u_c\left(c_t, 1-l_t\right) &= \beta \mathbb{E}_t \left[u_c\left(c_{t+1}, 1-l_{t+1}\right)\right. \\
&\quad \left.\left(\left(1-\tau_{kt+1}\right) r_{t+1}\mu_{t+1} + \left(1-\bar{\delta}\exp\left(a\left(\mu_{t+1}-\bar{\mu}\right)\right)\right)\right)\right]
\end{aligned}
$$

The first order conditions of the firm problem are:

$$
\begin{aligned}
r_t &= A_t F_{kt} \\
w_t &= A_t F_{lt}
\end{aligned}
$$

where F_{kt} and F_{lt} correspond to the marginal productivity of capital and labor respectively. r_t refers to the interest rate that was paid on the effectively used level of capital.

The equilibrium of this economy corresponds to a sequence of prices and quantities that satisfy the first order conditions and clear the markets. That is, given exogenous paths for efficiency, government expenditures, capital and labor wedges $\{A_t, g_t, \tau_{kt}, \tau_{lt}\}_{t=0}^{\infty}$, and $k_0 > 0$, the competitive equilibrium for $t \geq 0$ is defined as:

1. A sequence of factor prices $\{w_t, r_t\}_{t=0}^{\infty}$ and

2. A sequence of quantities $\{c_t, l_t, x_t, k_{t+1}, \mu_t\}_{t=0}^{\infty}$

such that:

1. The goods market is in equilibrium: $c_t + x_t + g_t = y_t$,

2. The factor markets are in equilibrium, that is, the excess demand for labor and capital are zero,

3. The government follows a balanced budget in every period according to which the purchases are financed with lump-sum taxes once the transfers to the households are taken into account, and

4. The first order conditions of the households and the firms are satisfied, and $k_{t+1} = \left(1-\delta_t\right) k_t + x_t$ holds.

The previous definition can be written compactly as follows:

$$
\frac{u_{lt}}{u_{ct}} = -\left(1-\tau_{lt}\right) A_t F_{lt} \tag{1}
$$

$$u_{ct} = \beta \mathbb{E}_t \left\{ u_{ct+1} \left[\left(1 - \tau_{kt+1}\right) A_{t+1} F_{kt+1} \mu_{t+1} + \left(1 - \bar{\delta} \exp\left(a\left(\mu_{t+1} - \bar{\mu}\right)\right)\right) \right] \right\} \tag{2}$$

$$\left(1 - \tau_{kt}\right) A_t F_{kt} = a \bar{\delta} \exp\left(a\left(\mu_t - \bar{\mu}\right)\right) \tag{3}$$

$$c_t + k_{t+1} + g_t = y_t + \left(1 - \bar{\delta} \exp\left(a\left(\mu_t - \bar{\mu}\right)\right)\right) k_t \tag{4}$$

$$y_t = A_t F\left(\mu_t k_t, \left(1 + \gamma\right)^t l_t\right) \tag{5}$$

Equations (1), (2), (4) and (5) represent, respectively, the labor supply, the Euler equation for consumption, the resource constraint of the economy and the production function. It can be seen how the wedges represent distortions that alter the decisions of agents in this economy. In particular, the labor wedge affects the relation between the marginal rate of substitution between consumption and leisure and the marginal productivity of labor. The capital wedge modifies the intertemporal relation between consumption and savings. The efficiency wedge influences the way in which the production factors are used in the production process. And the government wedge affects the availability of resources in the economy.

3 Correspondence between the benchmark and some detailed models

In the following it is shown how the wedges included in the benchmark model are related to some detailed models commonly used in the literature. Models that exhibit working capital constraints can be related to fluctuations in the benchmark's model efficiency and labor wedges. The costs associated with the existence of asymmetric information, like the ones in the financial accelerator literature, affect the consumption-saving decisions of the agents. Such distortion shows up in the benchmark model as a wedge on capital. Finally, the government wedge can be generated en detailed economies that have access to international markets to finance their deficits or borrow their surpluses.

3.1 Efficiency wedge

In general, it is possible to generate endogenous fluctuations in the total factor productivity (TFP) by using models with working capital constraints which generate an inefficient allocation of inputs in the production process. Such inefficiencies translate into shocks on aggregate productivity.

CKM show how in a closed economy with constant technology, production-inputs finance restrictions lead to an inefficient utilization of the productive factors. Such frictions in the detailed economy relate to the efficiency wedge of the benchmark model[3]. The same idea is used by Lama [2009] who uses the open economy model of Christiano et al. [2004] to show that movements in the TFP generated by changes in the world interest rate and in the prices of imported goods are equivalent to variations in the efficiency wedge of the benchmark model. In what follows, the derivations made by Lama [2009] are replicated.

Consider a small open economy where the final good producer has a technology of the following form:

$$q_t = \left(q_t^T\right)^{\phi} \left(q_t^{NT}\right)^{1-\phi}$$

where q_t^T and q_t^{NT} are the gross production in the tradable and non tradable sectors respectively. This agent faces the following problem:

$$\max_{q_t^T, q_t^{NT}} \quad q_t - p_t^T q_t^T - p_t^{NT} q_t^{NT}$$

where p_t^T and p_t^{NT} are the market prices of the intermediate production. The intermediate sectors produce with the following technology:

$$q_t^i = \left(m_t^i\right)^{\theta_i} \left(z_t^i\right)^{1-\theta_i} \qquad i = T,\ NT$$

where m_t^i and z_t^i are the quantities of imported inputs and value added in each sector. The amount of imported input in each of the sectors is constrained by a financing restriction. The problem of the agents in each sector is the following:

$$\max_{z_{it}, m_{it}} \quad p_t^i q_t^i - v_t z_t^i - R_t^* p_t^m m_t^i$$

where v_t, p_t^m and R_t^* are the price of the value added, the price of the imported inputs and the gross international interest rate respectively. The latter reflects the fact that producers must finance their access to inputs coming from international markets. Moreover, the firms producing value added, $z_t = z_t^T + z_t^{NT}$, try to maximize their profits:

$$\max_{k_t, l_t} = v_t k_t^{\alpha} l_t^{1-\alpha} - w_t l_t - r_t k_t$$

The GDP of this economy is defined as: $y_t = q_t - p_t^m \left(m_t^T + m_t^{NT}\right)$. In partial equilibrium, it is possible to show that the production function of the whole economy can be written as:

$$y_t = \Gamma\left(R_t^*, p_t^m\right) k_t^{\alpha} l_t^{1-\alpha}$$

Furthermore, as shown by Lama [2009], if it is assumed that $\theta_T = \theta_{NT} = \theta$, then the

[3] A detailed derivation of this proposition can be found in Bäurle and Burren [2008]

function $\Gamma(\cdot)$ is given by:

$$\Gamma(R_t, p_t^m) = \left[\phi^{\phi}(1-\phi)^{(1-\phi)}\theta^{\theta}\right]^{\frac{1}{1-\theta}}\left(\frac{1}{R_t^*}\right)^{\frac{\theta}{1-\theta}}\left(\frac{1}{p_t^m}\right)^{\frac{\theta}{1-\theta}}\left[1-\theta\left(\frac{\phi}{R_t^*}+\frac{1-\phi}{R_t^*}\right)\right]$$

according to which the allocations in this detailed economy and the allocations in the benchmark model are identical if the following condition is verified:

$$A_t = \Gamma(R_t^*, p_t^m) \tag{6}$$

3.2 Labor wedge

The model introduced in Section 3.1 also creates a labor wedge. That is, for the equilibrium allocations between the detailed and the benchmark model to be the same it must hold additionally that:

$$(1-\tau_{lt}) = \frac{A_t}{v_t} = \frac{\Gamma(R_t^*, p_t^m)}{v_t} \tag{7}$$

However, there are a number of different detailed models that can be represented by the benchmark model through time varying wedges. For example, Lama [2009] introduces a small open economy model based on the work of Neumeyer and Perri [2005] where restrictions on the way firms can finance their labor input for the production process are introduced.

In the following, the correspondence between the benchmark model and the equilibrium allocations from the limited participation model of Christiano and Eichenbaum [1992] is presented. A detailed derivation can be found in Bäurle and Burren [2008].

Let's consider an economy where the representative agent has to allocate Q_t monetary units out of its total available nominal balances, M_t, to consumption C_t. The excess, $M_t - Q_t$, is saved in the financial system which pays her a gross interest rate R_t. Consumption has to be entirely financed with Q_t and with the income derived from work:

$$Q_t + W_t L_t \geq P_t C_t$$

where W_t is the nominal wage. The agent faces the following budget constraint:

$$M_{t+1} = R_t(M_t - Q_t) + D_t + F_t + (Q_t + W_t L_t - P_t C_t)$$

where F_t and D_t are, respectively, the profits obtained from the firms and from the financial sector, which she owns. The agent seeks to maximize the discounted valued of the expected life time utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, L_t, H_t\right), \qquad \beta \in (0,1)$$

where H_t is a transaction cost related to the time spent in choosing Q_t. The main difference of this setup respect to *cash-in-advance* models, is that Q_t has to be chosen before the realization of the period t monetary shock.

On the other hand, the representative firm has to pay wages in cash, which he gets out from the financial system who charges him an interest rate R_t. Hence the cost of hiring a new employee is given by $R_t W_t L_t$.

Once the equilibrium conditions are derived it is possible to compare them with the ones in the benchmark economy what allow us to conclude that in order for them to be equivalent it must hold:

$$(1 - \tau_{lt}) = \frac{1}{R_t}\left(1 - \frac{H_t}{1 - L_t}\right) \tag{8}$$

3.3 Capital wedge

In general, the BCA literature has looked for correspondences between detail models and investment wedges. However, Christiano and Davis [2006] show that models like the financial accelerator proposed by Bernanke et al. [1999] create capital wedges instead[4].

Let's suppose that the economy is inhabited by households and entrepreneurs that offer, respectively, labor and capital to a final good producer. The entrepreneurs have a certain degree of expertise in the administration of capital. The capital is obtained from a producer that combines past capital stock and new investment to produce new capital. The income generated from the final good production is split in equal portions and distributed to the owners of the production factors. Households save part of their income in the financial system where they get an interest rate. Additionally, banks lend money to the entrepreneurs who prefer to leverage in order to accumulate more capital.

The friction in this model comes from the conflict of interests associated with the asymmetric information problem between the entrepreneurs and the banks. The banks don't know whether the entrepreneurs are doing well or not. They could be getting higher or lower profits than expected. Hence the only way for the former to find out what is happening at the interior of the entrepreneur business is to incur in a monitoring cost. As a result, the entrepreneurs have an incentive to manipulate the information they hand in to the banks with the purpose of getting higher loans.

This additional cost that banks have to pay is inversely related to the amount of self resources that the entrepreneurs use to finance their projects. That is, the higher the self

[4] An application of this model for an small open economy can be found in López et al., 2008

resources used in the economic activity, the lower the frictions in the economy. Following Lama [2009], banks have to be compensated for these monitoring costs according to the following relationship:

$$\left(1+R_{t+1}^{k}\right)=\left(1+\chi\left(\frac{N_{t+1}}{q_{t}K_{t+1}},\bar{\omega}_{t+1}\right)\right)(1+R_{t})$$

where N_{t+1}, q_t and R_t are the net worth of the entrepreneurs, the price of capital and the interest rate on the households' savings. $\bar{\omega}_{t+1}$ is a random variable related to the probability of default of the borrowers. The function $\chi(\cdot)$ is proportional to the leverage level of the entrepreneurs. Hence, $(1+\chi(\cdot))$ is the premium charged by the financial sector to the entrepreneurs. In other words, the previous relation measures the intermediation margin. Following Christiano and Davis [2006], the previous equilibrium relation can be stated in the following way:

$$\left(1+R_{t+1}^{k}\right)=(1-\tau_{k,t+1})(1+R_{t+1}) \tag{9}$$

where the premium charged to the entrepreneurs is an increasing function of the leverage and of the monitoring costs. Thus, the allocations of this detailed economy and those of the benchmark model are equivalent if equation (9) is verified.

3.4 Government wedge

CKM introduce a detailed model according to which it is possible to obtain a government wedge. Their derivations are replicated in what follows. Consider the world economy with a homogeneous good, where N countries can borrow and lend between them.

The equilibrium of this economy is characterized by a set of consumption, capital, labor and foreign debt allocations , $\left\{c_t^{i*},k_{t+1}^{i*},l_t^{i*},b_{t+1}^{i*}\right\}_{t=1,\ldots,\infty;i=1,\ldots,N}$, and a sequence of prices for the foreign debt, $\{q_t^{*}\}_{t=0}^{\infty}$, such that the household problem in each country i is solved and the world budget constraint is satisfied[5]:

$$\sum_{i=1}^{N}\left[c_t^{i*}+k_{t+1}^{i*}\right]\leq\sum_{i=1}^{N}\left[F\left(k_t^{i*},l_t^{i*}\right)+(1-\delta)\,k_t^{i*}\right]$$

The net exports of country i are given by:

$$XN_t^{i*}=F\left(k_t^{i*},l_t^{i*}\right)-\left(k_{t+1}^{i*}-(1-\delta)\,k_t^{i*}\right)-c_t^{i*}$$

The equilibrium allocations of this economy are equivalent to the closed economy benchmark model if the government wedge is defined as:

[5]Notice that in equilibrium the aggregate resource constraint the debt is zero.

$$g_t^i = XN_t^{i*} \tag{10}$$

If the detailed economy includes government consumption, then the government wedge in the benchmark model is defined as the sum of net exports and government consumption in the detailed economy.

The previous results are just some examples of the equivalences that can be derived to characterize the class of models that better explain the movements in the macroeconomic aggregates. Table 1 on page 12 provides a list of additional frictions that can be added to DSGE models and the corresponding wedge required to match with the benchmark model. Detailed derivations can be found in CKM, Bäurle and Burren [2008] and Lama [2009].

Table 1: Detailed economies and implicit wedges

	Wedge			
Friction	Efficiency	Labor	Capital	Government
Asset reallocation (Limited participation)			×	
Financial frictions (Limited participation)		×		
Labor hoarding	×			
Input financing distortions	×	×	×	
Adjustment cost on investment			×	
Open economy				×
Sticky wages		×		
Variable capital utilization	×		×	

Source: Bäurle and Burren [2008]

Finally, it is important to clarify that the wedges just derived assume similar functional forms both in the detailed economy and in the benchmark model. Otherwise, CKM show that the wedges will include the frictions of the detailed economy and the differences in preferences and technology specification. If both economies have different preferences and/or technologies it is necessary to identify just the impact of the distortion on the output fluctuations. This would allow us to direct in a proper way the economic intuition towards the relevant macroeconomic theories, avoiding possible wrong conclusions due to miss-specification problems.

4 Quantitative analysis

4.1 Calibration

Once the first order conditions of the benchmark model are obtained we proceed to calibrate both the functional forms and the parameters that define them. The following functional forms are chosen due to comparability with the literature on BCA. The preferences of the household are given by:

$$u\left(c_t, l_t\right) = \log c_t + \psi \log\left(1 - l_t\right) \tag{11}$$

where ψ is the relative weight of leisure in the utility function. On the other hand, technology is characterized by a Cobb-Douglas production function:

$$F\left(\mu_t k_t, l_t\right) = \left(\mu_t k_t\right)^{\alpha} \left(\left(1+\gamma\right)^t l_t\right)^{1-\alpha} \tag{12}$$

Hence, the first order conditions of the benchmark model are given by:

$$\psi \frac{c_t}{1-l_t} = \left(1-\tau_{lt}\right)\left(1-\alpha\right)\left(1+\gamma\right)^t A_t \left(\mu_t k_t\right)^{\alpha} \left(\left(1+\gamma\right)^t l_t\right)^{-\alpha} \tag{13}$$

$$\frac{1}{c_t} = \beta \mathbb{E}_t \left\{ \frac{1}{c_{t+1}} \left[\left(1-\tau_{kt+1}\right) \alpha A_{t+1} \left(\mu_{t+1} k_{t+1}\right)^{\alpha-1} \left(\left(1+\gamma\right)^{t+1} l_{t+1}\right)^{1-\alpha} \mu_{t+1} + \left(1 - \bar{\delta} \exp\left(a\left(\mu_{t+1} - \bar{\mu}\right)\right)\right) \right] \right\} \tag{14}$$

$$\left(1-\tau_{kt}\right) \alpha A_t \left(\mu_t k_t\right)^{\alpha-1} \left(\left(1+\gamma\right)^t l_t\right)^{1-\alpha} = a \bar{\delta} \exp\left(a\left(\mu_t - \bar{\mu}\right)\right) \tag{15}$$

$$c_t + k_{t+1} + g_t = y_t + \left(1 - \bar{\delta} \exp\left(a\left(\mu_t - \bar{\mu}\right)\right)\right) k_t \tag{16}$$

$$y_t = A_t \left(\mu_t k_t\right)^{\alpha} \left(\left(1+\gamma\right)^t l_t\right)^{1-\alpha} \tag{17}$$

The structural parameters are classified in two: those based on previous work in the subject and those calibrated in such a way that there exists consistency between the steady state of the model and the long run value of the observed macroeconomic variables. The long run value is defined as the historical average of the time series. The first group of parameters include the capital share, α, obtained from Parra [2008]; average depreciation rate, $\bar{\delta}$, and the discount factor, β, obtained from Bonaldi et al. [2009]; average capital utilization, $\bar{\mu}$, constructed as the historical average of the average between the capital utilization indices from Fedesarrollo and ANDI.

To calibrate the second group of parameters, we use the steady state of the benchmark model and the first set of parameters. In particular we are looking for a set of parameters (ψ and a) such that the agents devote, in average, 28% of the total time to work, number that is consistent with the data from the *Gran Encuesta Integrada de Hogares* completed by DANE (Colombian Statistics Office). Appendix A shows the steady state relations of the benchmark model. Table 2 on page 14 presents the parameter values used in the business cycle accounting exercise. In general, all the parameter values are consistent with the long run ratios of the Colombian economy, as measured from average values.

Table 2: Benchmark model parameters

Parameter		Value
Capital share	α	0.3600
Average depreciation rate	$\bar{\delta}$	0.0287
Discount factor	β	0.9864
Leisure participation in utility	ψ	1.5000
Capital use intensity	a	2.5000
Average capital utilization	$\bar{\mu}$	0.7300

Source: Parra [2008], Bonaldi et al. [2009], Banco de la República, ANDI and Fedesarrollo
All the rates are quarterly

4.2 Data

The data used in the business cycle accounting exercise correspond to the seasonally adjusted real per capita series of gross domestic product, households consumption, government consumption and gross investment from the 1994:Q1 to 2009:Q4 national account system computed by DANE. All in constant prices of 1994. Additionally, information on normally worked hours during the week in the seven main cities is used for the same period. This information is obtained from the matched series computed by the Technical and Economic Information Department from Banco de la Republica of the different household surveys introduced over the years by DANE. Finally, we use the capacity utilization indices from Fedesarrollo and ANDI.

Furthermore, to be able to proceed with BCA exercise we need a per capita capital stock series. To construct it, the law of aggregate capital accumulation is used:

$$K_{t+1} = (1 + \delta_t)\, K_t + X_t$$

Assuming a constant growth rate of population, γ_n[6], the per capita capital stock movement law is given by:

$$(1 + \gamma_n)\, k_{t+1} = (1 - \delta_t)\, k_t + x_t \tag{18}$$

The depreciation rate used in the computation is defined as:

$$\delta_t = \bar{\delta} \exp\left(a\left(\mu_t - \bar{\mu}\right)\right) \tag{19}$$

Once the depreciation rate is calculated, the per capita capital stock is constructed backwards in time assuming that in the last quarter of 2009 this variable was in steady state. The steady state of per capita capital is given by:

[6] It is assumed a quarterly growth rate of population of 0.33715%, that corresponds to the historical average of the growth rate calculated by DANE based on the latest population census.

$$k = \left(\frac{1}{\bar{\delta} + \gamma_n}\right) x \tag{20}$$

where x is the steady state of per capita gross investment, assumed to be the historical average. The evolution of all the time series used in the exercise are shown in Appendix B.

4.3 Computation of the wedges

Using the data from Section 4.2 we construct all of the wedges that distort the first order conditions of the benchmark model, A_t, τ_{lt}, τ_{kt} and g_t. Similarly, their behavior is studied by using in some cases the detailed models introduced in Section 3. Regarding this interpretations it is important to recall that the purpose of the BCA methodology is not to explain the evolution of the wedges, so all the possible explanations given here are just hypothesis or claims that need to be tested in further research.

The efficiency wedge (A_t) its constructed using the production function:

$$A_t = \frac{y_t}{(\mu_t k_t)^\alpha \left((1+\gamma)^t l_t\right)^{1-\alpha}} \tag{21}$$

The labor wedge (τ_{lt}) its calculated by means of the households labor supply equation, that is, by using the marginal rate of substitution between leisure and consumption:

$$\tau_{lt} = 1 - \frac{\psi c_t}{(1-l_t)(1-\alpha)(1+\gamma)^t A_t (\mu_t k_t)^\alpha \left((1+\gamma)^t l_t\right)^{-\alpha}} \tag{22}$$

The capital wedge (τ_{kt}) its obtained from the household first order condition with respect to the capital utilization :

$$\tau_{kt} = 1 - \frac{a\bar{\delta}\exp\left(a\left(\mu_t - \bar{\mu}\right)\right)}{\alpha A_t (\mu_t k_t)^{\alpha-1} \left((1+\gamma)^t l_t\right)^{1-\alpha}} \tag{23}$$

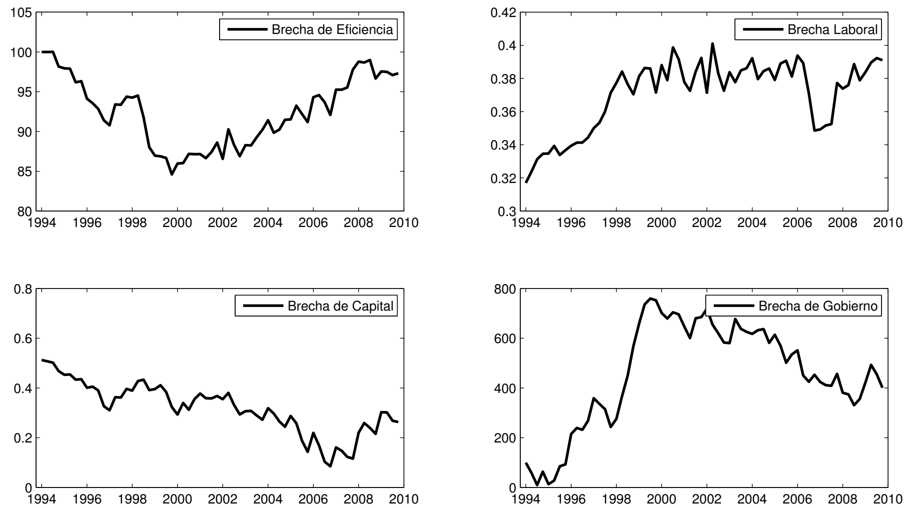
Finally, the government consumption wedge is derived from the aggregate resource constraint of the economy:

$$g_t = y_t - c_t - x_t \tag{24}$$

Note that under identity (24), the government wedge used in this exercise includes both net exports and government consumption.

The method used to construct the wedges is different from the proposal of CKM. By introducing the variable capital utilization in the benchmark model, the four wedges can be calculated using only information of period t without using the Euler equation of consumption. Hence, there is no need to calculate the expectations of future realizations of some macroeconomic variables or wedges in their calculation. The derived wedges are plotted in Figure 1 on page 16, where the efficiency and government wedges were normalized to their first value.

Figure 1: Benchmark model wedges: 1994:Q1 - 2009:Q4



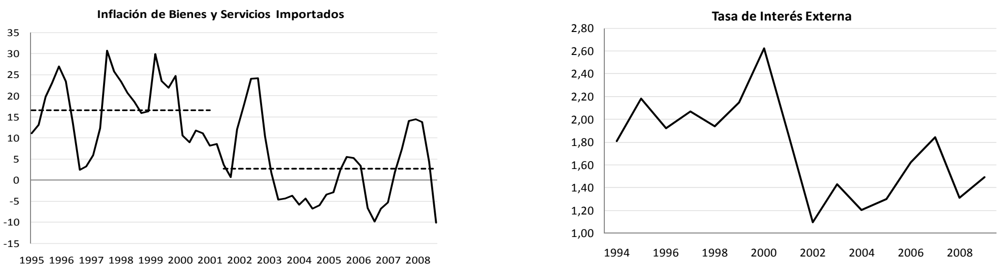
Source: Own calculations

Between 1994 and 2009 the efficiency wedge behaves in two opposite ways. Up 1999, the pronounced fall in the wedge can be explained by the economic liberalization that began with the introduction of the 1991 National Constitution, and the economic downturn of the end of that decade caused by the Asian crisis. After 1999, the TFP shows a significant increase that takes it to similar values of those of late 1990 [7]. This behavior is consistent with the predictions from Christiano et al. [2004] introduced in Section 3.1. As can be seen in Figure 2 on page 17 the inflation of imported goods and services shows two levels or regimes along all the period. An average inflation of 17% before 2002 and an average inflation of 3% between 2002 and 2009. The international interest rate exhibits a similar behavior [8].

[7] The evolution of the efficiency wedge is similar to the residual obtained from a econometric regression of the Cob-Douglas production function: Solow residual.

[8] The international interest rate corresponds to the most representative rate for Colombian firms willing to borrow in foreign markets calculated by the Technical and Economic Information Department of Banco de la República.

Figure 2: Imports inflation and external interest rate (%)

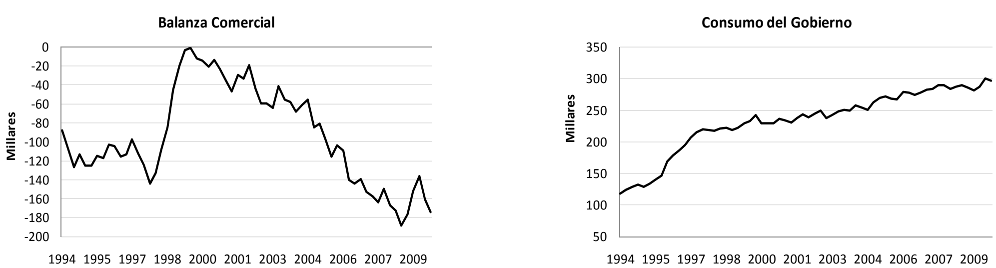


Source: DANE and own calculations

Source: Banco de la República

On the other hand, the government wedge shows a continuous increase from 1996 until mid 2000, followed by a slow decline. By construction, its behavior can be explained by both the government consumption and the net exports. Figure 3 on page 17 allow us to conclude that their movements are dominated by those of net exports which are amplified by the government expenditures mainly between 1995 and 1998. Hence, the registered increase in the wedge during the first part of the sample is explained by a reduction in the trade balance deficit and the higher government consumption. After the 1999 crisis, the wedge begins to fall due to the worsening in the trade balance. However, its fall is not too pronounced thanks to the stability of the government spending.

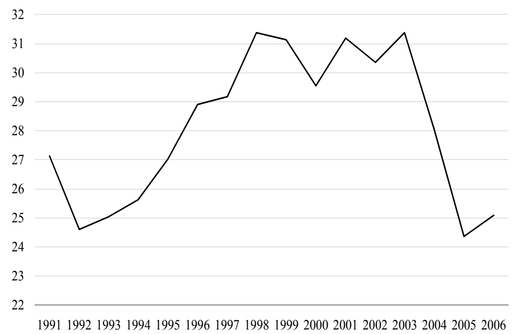
Figure 3: Government consumption and net exports



Source: DANE

Regarding the capital wedge, we observe a constant decrease along the sample. This behavior is consistent with the predictions of the financial accelerator model of Bernanke et al. [1999]. As mentioned in Section 3.3 this wedge can be related with the finance cost of capital, which at the same time is a function of the economy's leverage level. (see equation 9). Hence, the greater the leverage, the greater the premium that the financial sector charges to the agents, a situation that coincides with the evolution of the capital wedge in Colombia following the evidence provided by López and Rodríguez [2008]. According to the authors, during the five year previous to the 1999 crisis the economy registered a constant increase in

Figure 4: Leverage (%)



Source: López and Rodríguez [2008]

the leverage [9] (Figure 4 on page 18). This behavior can be explained by an increased trust from households and firms regarding future profits due to a greater availability of credits after the financial liberalization of the early 90's.

Finally, the labor wedge shows a relatively stable behavior after the 1999 crisis. However between 1994 and 1999 it registers a constant expansion. This phenomenon coincides with the operational start of the new social security system introduced by the Law 100 of 1993, that raised the labor cost both to firms and employees [10]. Similarly, the behavior of the domestic interest rate, measured by the interbank rate (TIB), could explain the evolution of the wedge at the beginning of the sample. According to Section 3.2, a high nominal interest rate can amplify the distortion in the labor supply relation from the household when there are input financing restrictions. Figure 5 on page 19 shows the level of TIB for the entire time period, which can be characterized by two regimes defined by the beginning of the inflation targeting scheme of the Central Bank in the late 90's. Hence, a reason for the later stabilization of the labor wedge could be based on Christiano and Eichenbaum [1992], that is, a Central Bank reduction and stabilization of the interest rate.

4.4 Business cycle accounting in Colombia

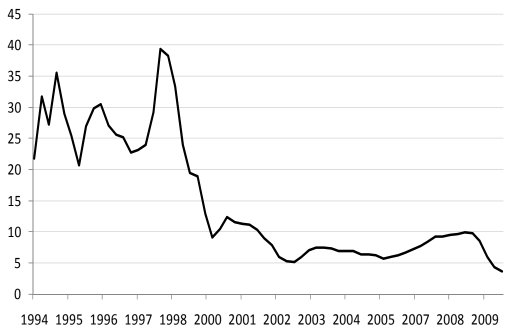
4.4.1 Simulation with individual wedges

Following the proposal of CKM, once the four wedges are constructed we proceed to assess the relative importance of each of them in the observed movements of the main macroeconomic aggregates. To do so, different simulation exercises of the benchmark model are done by introducing the individual wedges, or combinations of them, and leaving the remaining wedges

[9] The leverage is measured by the authors as the ratio between total nonperforming loans and capital stock.

[10] For a discussion on the subject and its possible effects on unemployment in Colombia, see Arango and Posada [2001]

Figure 5: Domestic interest rate (%)

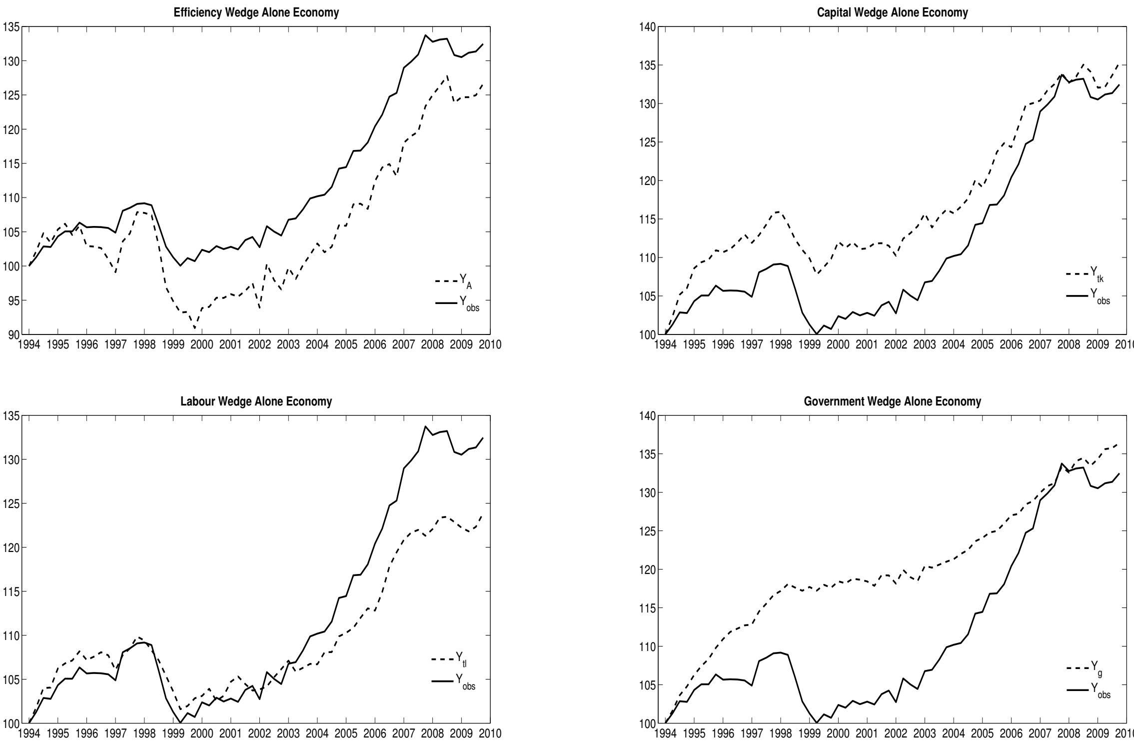


Source: Banco de la República

in their mean or long run value. As described by Cavalcanti et al. [2008] the aim is to find the paths for output (y_t), consumption (c_t), working hours (l_t) and fraction of capital used in production (μ_t) that result from the solution of the non linear system formed by the first order conditions of the model (equations (21) to (24)) once the observed values of capital stock and wedges are introduced. The exercise can be extended to include investment as one of the simulated variables. However, as shown in Mahadeva and Parra [2008] its volatility results in general in a loss of fit of DSGE models. By construction, when the four wedges are introduced simultaneously, the simulated series are equivalent to the observed series.

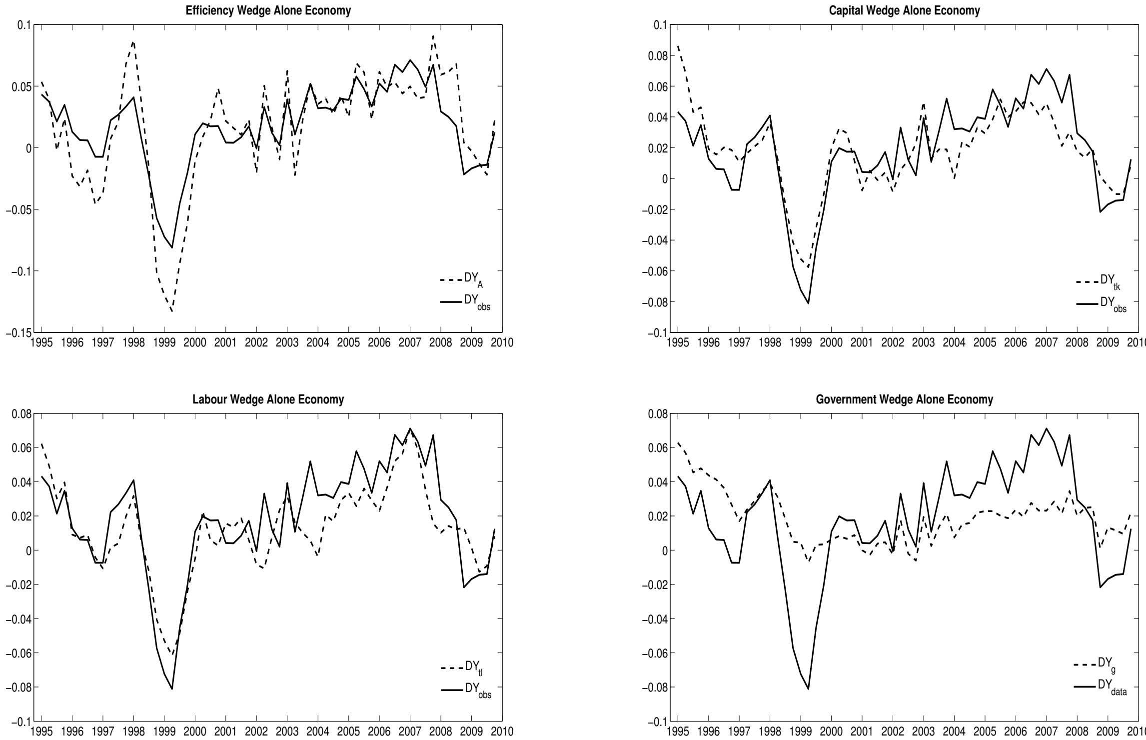
Figure 6 on page 20 and Figure 7 on page 21 show the simulations for the GDP levels and annual growth rates when just one wedge is introduced. The continuous lines represent the observed values while the dotted lines represent the implicit series for each of the simulated models. The results for consumption, hours worked and capital utilization are presented in Appendix C. By just doing a graphical analysis it is possible to conclude that the efficiency wedge, the labor wedge and the capital wedge capture correctly the movements in the levels of GDP. However, when the annual growth rates are analyzed, the efficiency wedge losses fit since it tends to overestimate the 1999 economic crisis, while the other two wedges, labor and capital, predict better this event. On the other hand, the government wedge does not exhibit a great influence on GDP. In particular, the simulated path in levels overestimate constantly the observed GDP fluctuations and it is not able to account, in general, for the downturns and increases along the time period.

Figure 6: Impact of the individual wedges on the level of GDP



Source: Own calculatios

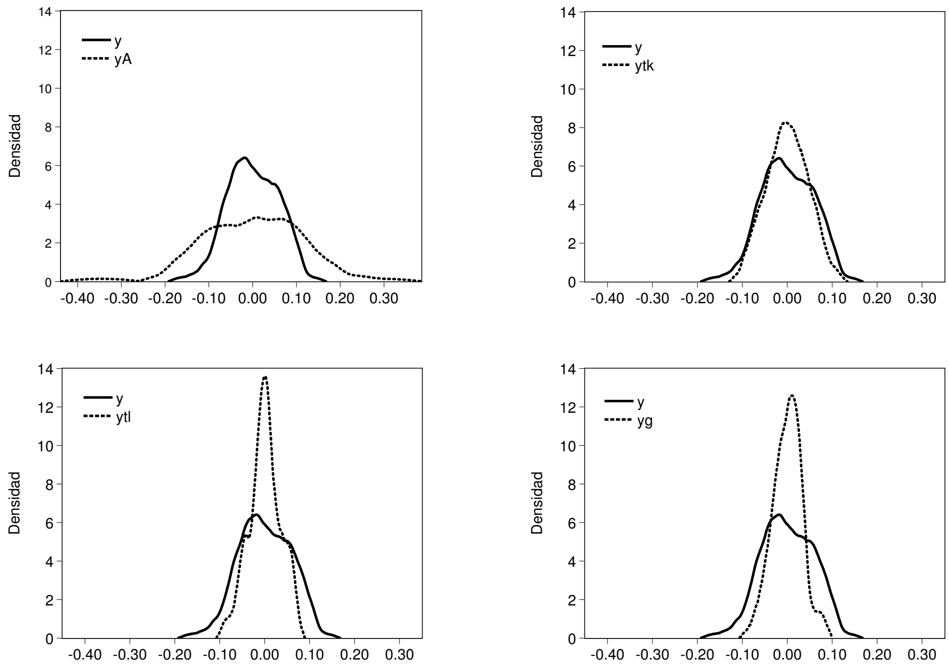
Figure 7: Impact of individual wedges on the annual growth of GDP



Source: Own calculations

To evaluate the goodness of fit of the different models we construct different measures that allow us to confirm the previous conclusions. The first exercise tries to characterize the probability distribution of the predictions of each of the models. To achieve this objective we estimate the density functions of each of the simulated series and compare it with the distribution of the observed GDP . The approximations are made by means of a Kernel density estimator described in Appendix D. This exercise is done only on the (covariance stationary) annual growth simulations in order to obtain consistent estimators of the density functions. Figure 8 on page 22 show the results.

Figure 8: Empirical distributions: annual growth rates of observed GDP vs. annual growth rates of implicit GDP



Source: Own calculations

From the four models, the capital wedge exhibits the best fit in terms of the probability distribution. It predicts a higher proportion of growth rates similar to the ones observed. This result differs from those obtained in similar studies for Latin-America that apply the CKM method. For example, for a group of Latin-American economies, included Colombia, Lama [2009] concludes that the frictions that alter the consumption-saving decision of the household are not useful to predict economic downturns. Simonovska and Söderling [2008] and Graminho [2006] find that while the efficiency and labor wedges play an important role to explain economic activity in Chile and Brazil respectively, the investment wedge is not significant. Similar conclusions have been obtained by other authors for the US case during the Great Recession and the Great Moderation. However, the goodness of fit from the capital wedge suggest that models like the ones proposed by Bernanke et al. [1999], and applied in Colombia by López and Rodríguez [2008] and López et al. [2008] can be relevant to explain the dynamics of the economy. Other examples of detailed models that can distort the Euler equation through imperfections in the financial markets are those of Bernanke and Gertler [1989], Carlstrom and Fuerst [1997] and Kiyotaki and Moore [1997].

Based on the probability distributions, the efficiency wedge shows the poorest fit given the longer tails of the distribution relative to those of observed GDP. Fluctuations in TFP can be generated through the model detailed in Section 3.1 or alternatively following the ideas of

Kehoe and Ruhl [2009] and Mendoza [2006].

The estimated distributions of the "labor wedge alone economy" and "government wedge alone economy" simulated GDP series are similar. They exhibit a leptokurtic behavior that implies a higher fraction of predictions around the 0% level, which reduces the probability of crisis or boom predictions in the economic activity. One alternative way to generate distortions in the household's labor decisions to the model presented in Section 3.2 can be found in the work of Cole and Ohanian [2004] and Mulligan [2002] who built detailed models with labor distortions through changes in the labor regulations.

Table 3 on page 23 add to the previous results by evaluating the goodness of fit of each the models in terms of their variability. The first measure is the traditional R^2 used for linear models. Additionally, we construct the ratio of volatilities between $\hat{y}_i$ and y, where $\hat{y}_i$ is the projected path by model i, with $i = A, \tau_k, \tau_l, g$: a ratio less than one means that the simulated model exhibits lower variability than the observed GDP, and a ratio greater than one the opposite. Finally, we construct Theil's U coefficient. This measure is bounded between 0 and 1, where 0 indicates the maximum precision possible. All the previous measures are applied to both simulated series, the levels of GDP and its annual growth rates.

Table 3: Goodness of fit

	Criteria					
GDP prediction	R^2		$\sigma_{\hat{y}_i}/\sigma_y$		$U-Theil$	
	Level	Annual growth	Level	Annual growth	Level	Annual growth
Efficiency wedge ($\hat{y}_A$)	0.9049	0.4296	**0.9695**	1.4805	0.1580	0.4562
Labor wedge ($\hat{y}_{\tau_l}$)	0.8550	0.7028	0.6456	0.8103	0.1978	0.2966
Capital wedge ($\hat{y}_{\tau_k}$)	**0.9283**	**0.7264**	0.8107	**0.8189**	**0.1364**	**0.2824**
Government wedge ($\hat{y}_g$)	0.7307	0.2493	0.7406	0.4723	0.2798	0.5779

Source: Own calculations

In general, the capital wedge explains more of the volatility observed in GDP, both in levels and annual growth rates. The models with efficiency wedges and labor wedges produce mixed conclusions. The model with efficiency wedge helps to predict the variability in the levels of GDP, while model with labor wedge it is better at explaining annual growth rates. Finally, the government consumption wedge generates poor predictions of GDP since it only explains 73% of the variability of output in levels and 24% of annual growth rates variability.

4.4.2 Simulation with combination of wedges

Given that the economy is constantly hit by different types of shocks, it is important to assess the goodness of fit of the model when it is fed with combinations of the wedges. The analysis of the previous section suggest the design of models that affect the link between the marginal rate of substitution between consumption and leisure and the marginal productivity of labor, the consumption-saving decision of the households, or the efficiency with which the productive factor are combined in the production process.

To give an initial idea of the possible relations between the different wedges we build a matrix of correlations between the different simulated paths of the previous section using Spearman's monotonic correlation coefficient. Results are shown in Table 4 on page 24. The numbers in the upper rows are associated with simulated levels, while the lower row values relate to the annual growth rates. All the coefficients are statistically different from zero[11].

Table 4: Spearman's correlation between implicit paths

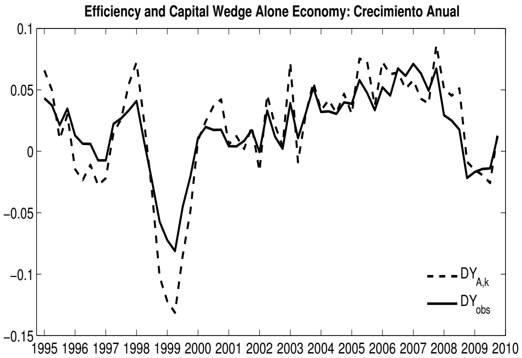
	$\hat{y}_A$	$\hat{y}_{\tau_l}$	$\hat{y}_{\tau_k}$	$\hat{y}_g$
$\hat{y}_A$	1.0000	**0.9193**	0.7480	0.5594
	1.0000	0.6962	0.7041	0.5558
$\hat{y}_{\tau_l}$	**0.9193**	1.0000	0.8800	0.6853
	0.6962	1.0000	**0.8837**	0.5630
$\hat{y}_{\tau_k}$	0.7480	0.8800	1.0000	0.8981
	0.7041	**0.8837**	1.0000	0.6969
$\hat{y}_g$	0.5594	0.6853	0.8981	1.0000
	0.5558	0.5630	0.6969	1.0000

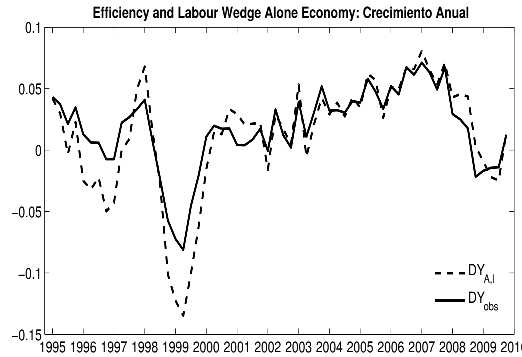
Source: Own calculations

It is important to note the high correlation (88.37%) between the predicted GDP that results from the "labor wedge alone economy" and the "capital wedge alone economy". This suggests that whenever the distortion in the labor supply increases, the distortions in the consumption-savings relation is expected to increase as well and vice versa. Hence, it is relevant to study combinations of wedges since it may be possible to obtain a better fit of the model. Figure 9 on page 25 presents the results from these simulations for the annual growth rates of the economy activity.

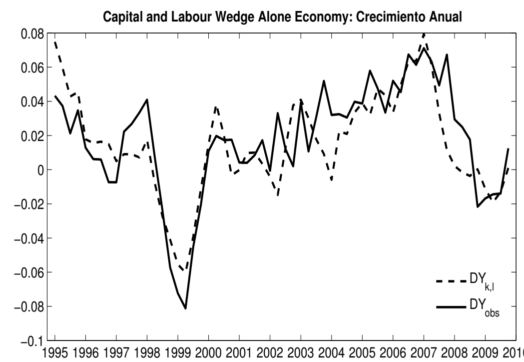
[11] Spearman's correlation coefficient is a non-parametric measure of statistical dependence defined as $\rho = 1 - \frac{6\sum d_i^2}{T\left(T^2-1\right)}$ where $d_i = x_i - y_i$ is the rank difference between the two variables x and y, and T is the sample size. The statistical significance of ρ it is evaluated with the statistic $t = \rho\sqrt{\frac{T-2}{1-\rho^2}} \sim t_{(T-2)}$.

Figure 9: Combined effects on the annual growth of GDP





R^2	62.51%	R^2	56.07%
$U - Theil$	0.3419	$U - Theil$	0.3790



R^2	64.57%
$U - Theil$	0.3300

Source: Own calculations

From the results, it is possible to conclude that detailed models that include frictions in the labor supply and in the Euler equation for consumption outperform other models with different combinations. This can be verified by analyzing the estimated densities which are presented in Appendix E. However, it is important to highlight the fact that the predictions obtained with individual wedges are not outperformed by the models with combinations of them.

5 Conclusions

In this document we apply the business cycle accounting procedure in order to study which mechanisms are more relevant to interpret the dynamics of the Colombian GDP between 1994 and 2009. These mechanisms can be identified through their effects on different types of distortions, called wedges, included in the first order conditions of the neoclassical growth model. The methodology is useful for macroeconomists as an alternative identification tool when designing detailed DSGE models, since it allows them to assess the importance of each of those mechanisms in the economic fluctuations.

The benchmark model used in this article differs from the original model proposed by CKM by introducing as endogenous variable in the model the variable capital utilization which at the same time simplifies the computation of the wedges. Furthermore, the wedge on investment is replaced by a wedge on capital. Under these assumptions, the computation of the wedges can be done by means of the non linear first order conditions avoiding the need for linear approximations that can be affected by the critics of Christiano and Davis [2006] or the identification conditions outlined by Bäurle and Burren [2007]. The accounting procedure is done in two steps: (i) information about output, consumption, capital stock, investment, hours worked and capacity utilization is used together with the first order conditions of the benchmark model to compute the time series for the wedges. This wedges measure how far is the observed economy from one closed, perfect competitive economy. The wedges can be interpreted as taxes on capital, labor income, efficiency shocks and exogenous movements in government expenditures. (ii) once the wedges are obtained, the benchmark model is simulated by feeding individual distortions or combinations of them in order to assess their relative importance in explaining the observed movements in GDP. Finally, the equivalence result that shows how the benchmark model is a reduced form representation of a great number of detailed DSGE models, can be applied.

The results suggest that in order to study the observed GDP fluctuations in Colombia between 1994 and 2009, macroeconomic models should include distortions related to the labor wedge or the capital wedge. That is, it is possible to get better predictions with models that introduce alterations on the labor supply or the Euler equation for consumption. The importance of the capital wedge contrasts with previous results found in emerging economies, except Argentina. According to Cavalcanti et al. [2008], this can be explained either by very particular conditions of the economy being studied or because the benchmark model used here does not rely on linearization procedures in order to construct the wedges.

The efficiency wedge could also be used to explain the behavior of GDP during economic stable periods but not to predict downturns or economic booms. In fact, while the first two wedges predict the 1999 crisis, the "efficiency alone economy" model overestimates this event. For Bäurle and Burren [2008], this lower ability of the efficiency wedge to explain movements in GDP is a sufficient condition to reject the RBC hypothesis according to which it is possible to model the economy only with technology shocks. The only class of models that it is possible to reject according to the results is that related to government wedges.

Finally, a high correlation between the different wedges is found. Future research, based on linear approximations, should focus on their study and their determinants. This way, it would be possible to constrain the spectrum of detailed models that are relevant to study business cycles. At the same time, the use of linear approximations would allow for the use of tools like the Kalman Filter in order to obtain a historical decomposition of shocks and the maximum likelihood estimation of the deep parameters of the model.

A Steady state

Using the equilibrium conditions of the benchmark model (13) to (17) the following steady state relations can be derived.

$$r = \frac{1 - \beta\left(1 - \bar{\delta}\right)}{\beta\left(1 - \tau_k\right)\bar{\mu}} \tag{25}$$

$$\frac{x}{k} = \bar{\delta} \tag{26}$$

$$\frac{y}{k} = \frac{r\bar{\mu}}{\alpha} \tag{27}$$

$$\frac{l}{k} = \left[\frac{1}{A\mu^{\alpha}\left(1+\gamma\right)^{1-\alpha}}\left(\frac{y}{k}\right)\right]^{\frac{1}{1-\alpha}} \tag{28}$$

$$w = \left(1 - \alpha\right)\frac{y}{l} \tag{29}$$

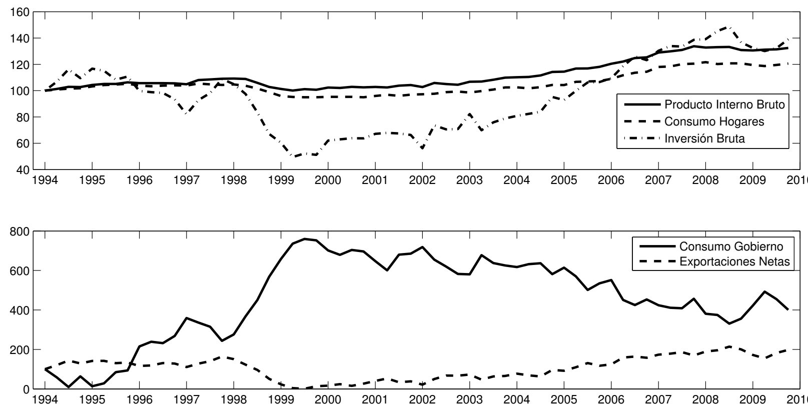
$$\psi\frac{\left(\frac{y}{l} - \frac{x}{l} - \frac{g}{l}\right)}{\left(1 - \tau_l\right)w} - \frac{1}{l} + 1 = 0 \tag{30}$$

$$\frac{c}{l} = \frac{y}{l} - \left(\frac{x}{k}\right)\left(\frac{k}{l}\right) - \frac{g}{l} \tag{31}$$

B Data

Figure 10 on page 29 shows the evolution of the main macroeconomic aggregates used in the accounting procedure for the period 1994 - 2009.

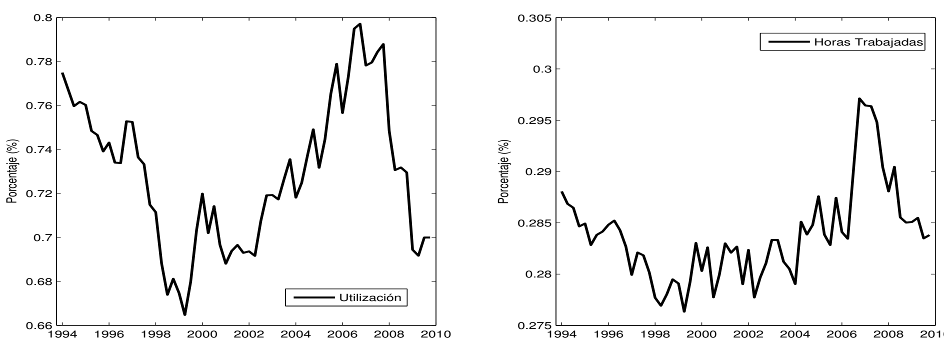
Figure 10: Main macroeconomic aggregates 1994:Q1 - 2009:Q4.



Source: DANE

On the other hand, Figure 11 on page 29 presents the behavior of the capacity utilization index and the hours worked for the same period.

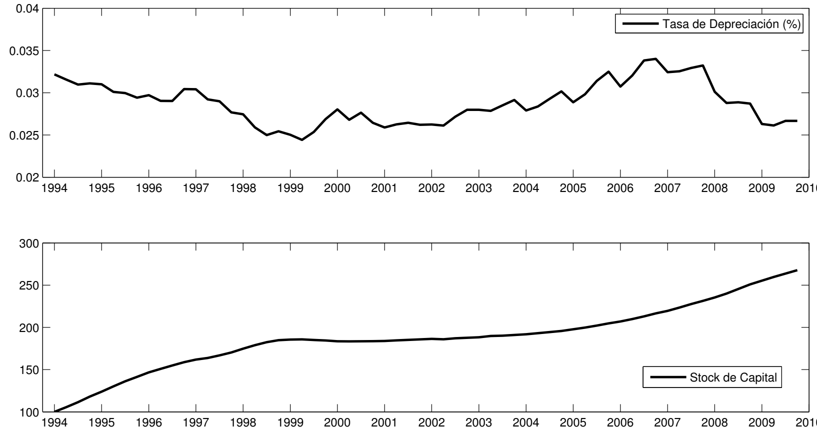
Figure 11: Capacity utilization and hours worked 1994:Q1 - 2009:Q4.



Source: DANE, ANDI, Fedesarrollo

Figure 12 on page 30 shows the resulting time series after the computation of the capital stock and its depreciation. Regarding the former, the result is similar to that obtained by Mahadeva and Parra [2008] using a different technique that aggregates three different capital stocks, each of them with its own depreciation rate.

Figure 12: Depreciation and Capital stock: 1994:Q1 - 2009:Q4



Source: Own calculations

C Simulations of consumption, hours worked and capital utilization

Figure 13 on page 31 to Figure 15 on page 32 show the predictions for consumption, hours worked and capital utilization obtained from the models fed with only one wedges at a time. For consumption, the annual growth rates are shown, while for hours worked and utilization of capital the levels are used instead.

Figure 13: Annual growth of consumption

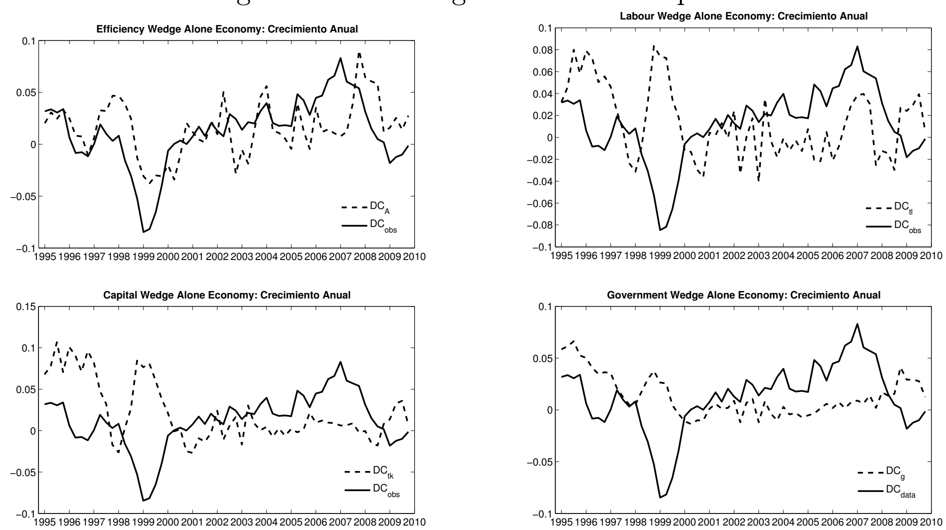


Figure 14: Hours worked

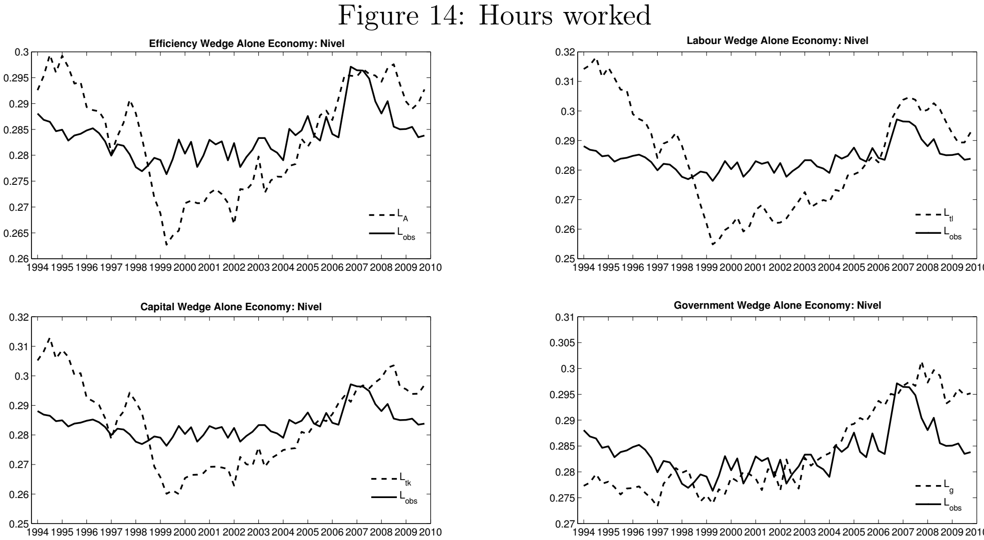
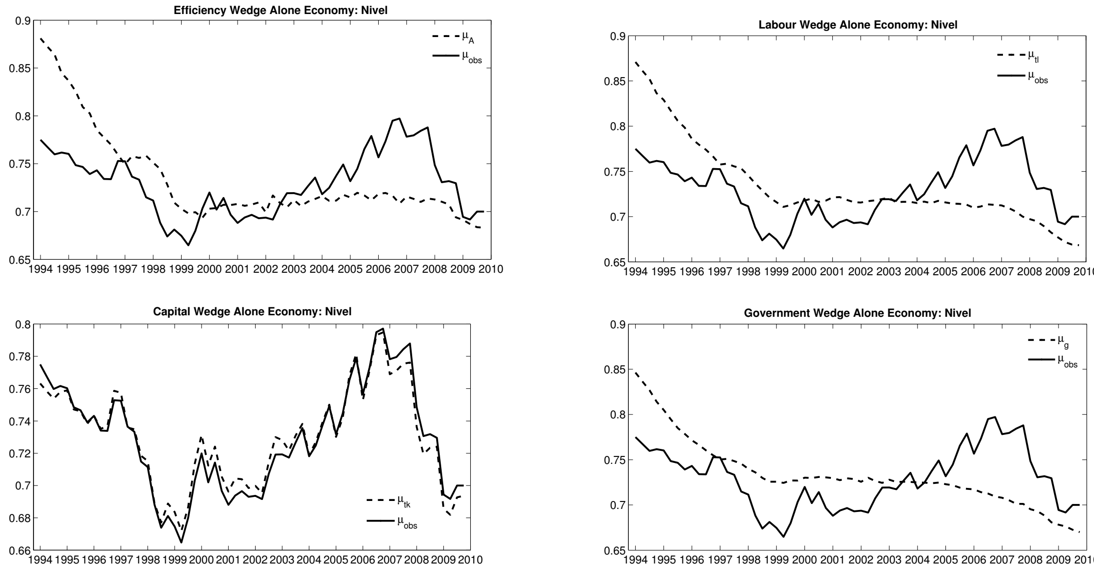


Figure 15: Capacity utilization



D Rosenblatt-Parzen Kernel Estimator

The Rosenblatt-Parzen Kernel estimator is a non-parametric estimator that avoids the need for assumptions regarding the true distribution functions of the random variables and delivers a smooth estimation of their density function.

Let $x_1, x_2, \ldots, x_T$ a sample from a random variable with probability distribution $f(x)$. The Rosenblatt-Parzen estimator is defined as:

$$\hat{f}(x) = \frac{1}{Th} \sum_{t=1}^{T} K\left(\frac{x_t - x}{h}\right) \tag{32}$$

where $K(\cdot)$ is a Kernel function such that $\int_{-\infty}^{\infty} K(\psi)\, d\psi = 1$.

h is the estimation smoothing parameter, also called band width, which depends on the sample size. For the estimation, the Epanechnikov Kernel defined as:

$$K(\psi_t) = \frac{3}{4}\left(1 - \psi_t^2\right) \mathbf{1}_{\{|\psi_t| \leq 1\}} \tag{33}$$

is used, where $\mathbf{1}_{\{\ldots\}}$ is an indicator function.

For the band width selection, we follow the proposal of Silverman [1986] which is based on the sample size:

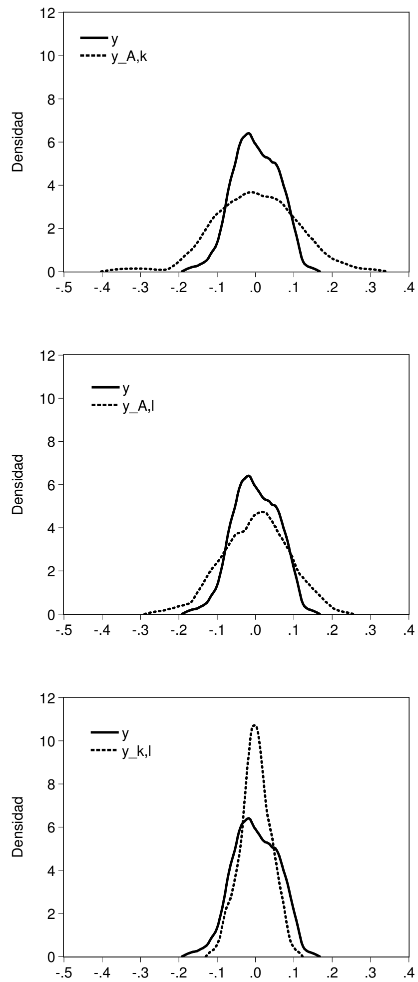
$$h = 0.9kT^{-\frac{1}{5}} \min\left\{s, \left(\frac{IQR}{1.34}\right)\right\} \tag{34}$$

where s is the sample standard deviation, IQR is the interquantile rank of the series and k is a scale factor that depends on the Kernel function used in the estimation.

The Kernel estimator exhibits desirable statistical properties whenever the sample is random, that is, when the observations are *iid*, which does not happen in this time series context. However, the estimator keeps their MCE consistency, weak consistency and asymptotic normality if the series studied are strictly stationary. Hence, the Kernel estimator is only used to assess the density function of the annual growth rates of realized and simulated GDP. A formal proof of the previous statement is found in Pagan and Ullah [1999].

E Estimated distributions: combination of wedges

Figure 16: Empirical distributions using Kernel estimation



Source: Own calculations

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